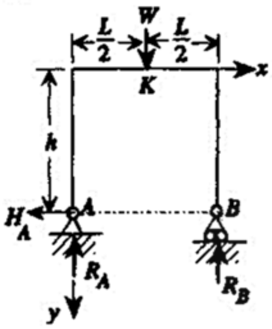
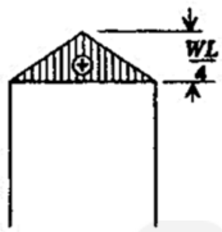
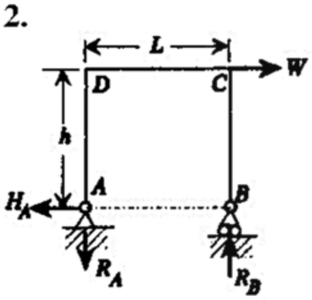
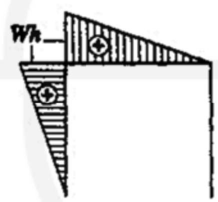
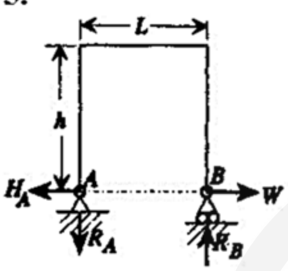
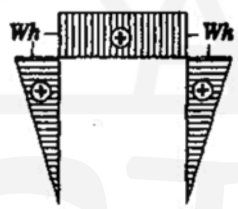
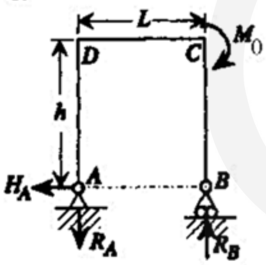
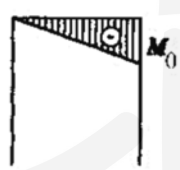
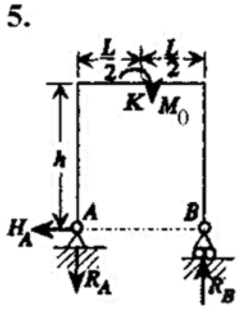
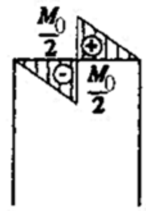
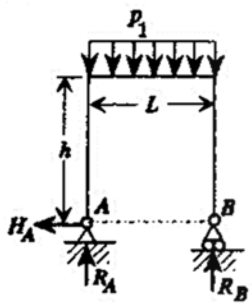
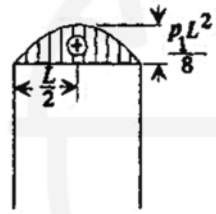
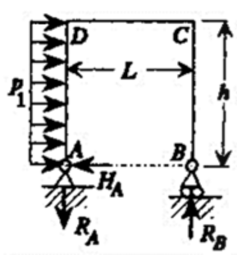
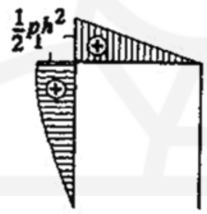
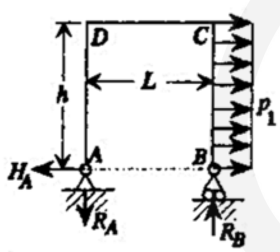
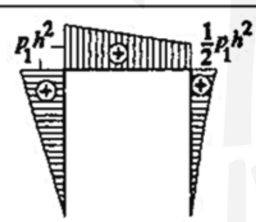
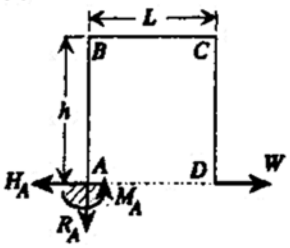
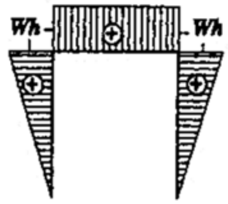
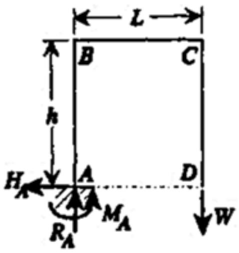
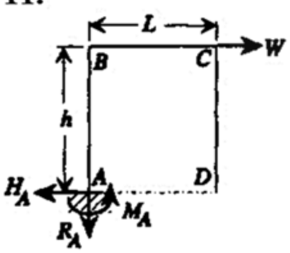
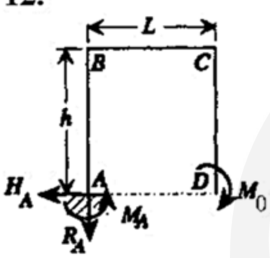
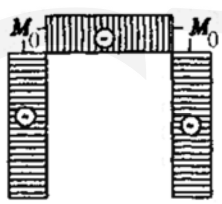
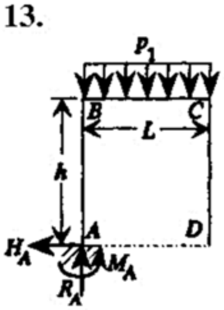
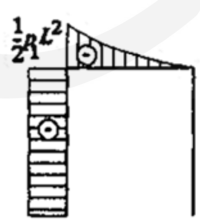
### 7.2 Bending Moment Diagrams and Equations for Frames

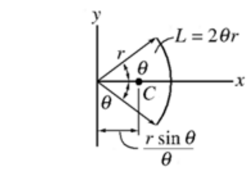
| Configuration | Moment Diagram | Important Values |
|---|---|---|
| **1.**   |   | $H_A = 0$<br>$R_A = R_B = \frac{1}{2}W$<br>$v_{Bx} = \dfrac{WhL^2}{8EI}$<br>$M_{\max} = \frac{1}{4}WL$ at point $K$ |
| **2.**   |   | $H_A = W$ $\quad R_A = R_B = W\dfrac{h}{L}$<br>$v_{Bx} = \dfrac{Wh^2}{6EI}(3L + 2h)$<br>$v_{Cy} = 0 \quad v_{Cx} = \dfrac{Wh^2}{3EI}(L + h)$<br>$M_{\max} = Wh$ at point $D$ |
| **3.**   |   | $H_A = W \quad R_A = R_B = 0$<br>$v_{Bx} = \dfrac{Wh^2}{3EI}(3L + 2h)$<br>$M_{\max} = Wh$ |
| **4.**   |   | $H_A = 0 \quad R_A = R_B = \dfrac{M_0}{L}$<br>$v_{Bx} = \dfrac{M_0 hL}{2EI}$<br>$M_{\max} = M_0$ at point $C$ |

| Configuration | Moment Diagram | Important Values |
|---|---|---|
| **5.**  |  | $H_A = 0 \quad R_A = R_B = \dfrac{M_0}{L}$<br>$\theta_K = \dfrac{M_0 L}{12EI}$<br>$M_{\max} = \frac{1}{2}M_0$ at point $K$ |
| **6.**  |  | $H_A = 0 \quad R_A = R_B = \frac{1}{2}p_1 L$<br>$v_{bx} = \dfrac{p_1 h L^3}{12EI}$<br>$M_{\max} = \frac{1}{8}p_1 L^2$ at $x = \frac{1}{2}L$ |
| **7.**  |  | $H_A = p_1 h \quad R_A = R_B = \dfrac{p_1 h^2}{2L}$<br>$v_{Bx} = \dfrac{p_1 h^3}{24EI}(6L + 5h)$<br>$M_{\max} = \frac{1}{2}p_1 h^2$ at point $D$ |
| **8.**  |  | $H_A = p_1 h \quad R_A = R_B = \dfrac{p_1 h^2}{2L}$<br>$v_{Bx} = \dfrac{p_1 h^3}{24EI}(18L + 11h)$<br>$M_{\max} = p_1 h^2$ at point $D$ |

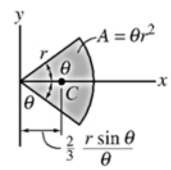
| Configuration | Moment Diagram | Important Values |
|---|---|---|
| **9.**  |  | $H_A = W \quad R_A = 0 \quad M_A = 0$<br>$v_{Dx} = \frac{Wh^2}{3EI}(3L + 4h)$<br>$v_{Dy} = -\frac{WhL}{2EI}(L + h)$<br>$M_{\max} = Wh$ at points $B$, $C$ |
| **10.**  |  | $H_A = 0 \quad R_A = W \quad M_A = WL$<br>$v_{Dx} = -\frac{WhL}{2EI}(L + 2h)$<br>$v_{Dy} = \frac{WL^2}{3EI}(L + 3h)$<br>$M_{\max} = WL$ |
| **11.**  |  | $H_A = W \quad R_A = 0 \quad M_A = Wh$<br>$v_{Dx} = -\frac{Wh^3}{2EI} \quad v_{Dy} = \frac{WLh^2}{2EI}$<br>$v_{Cx} = \frac{Wh^3}{3EI} \quad v_{Cy} = \frac{WLh^2}{2EI}$<br>$M_{\max} = Wh$ at point $A$ |
| **12.**  |  | $H_A = 0 \quad R_A = 0 \quad M_A = M_0$<br>$v_{Dx} = \frac{M_0 h}{EI}(L + 3h)$<br>$v_{Dy} = -\frac{M_0 L}{2EI}(L + 2h)$<br>$\theta_D = \frac{M_0}{EI}(L + 2h) \quad M_{\max} = M_0$ |
| **13.**  |  | $H_A = 0 \quad R_A = p_1 L$<br>$M_A = \frac{1}{2} p_1 L^2$<br>$v_{Dx} = -\frac{p_1 L^2 h}{6EI}(L + 3h)$<br>$v_{Dy} = \frac{p_1 L^3}{8EI}(L + 4h)$<br>$M_{\max} = \frac{1}{2} p_1 L^2$ |

## 7.3 Geometric Properties of Line and Area Elements:

Centroid Location | Centroid Location | Area Moment of Inertia



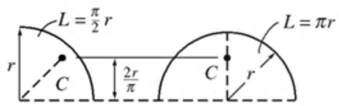

Circular arc segment



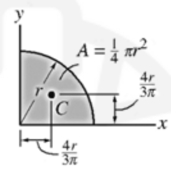

Circular sector area

$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2} \sin 2)\theta$

$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2} \sin 2)\theta$



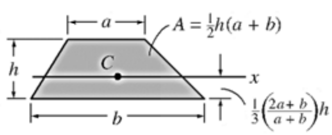

Quarter and semicircle arcs



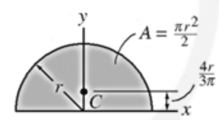

Quarter circle area

$I_x = \frac{1}{16} \pi r^4$

$I_y = \frac{1}{16} \pi r^4$



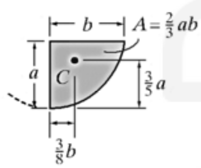

Trapezoidal area



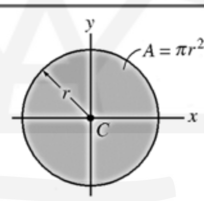

Semicircular area

$I_x = \frac{1}{8} \pi r^4$

$I_y = \frac{1}{8} \pi r^4$



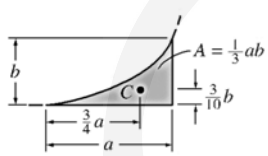

Semiparabolic area



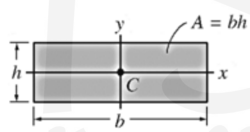

Circular area

$I_x = \frac{1}{4} \pi r^4$

$I_y = \frac{1}{4} \pi r^4$



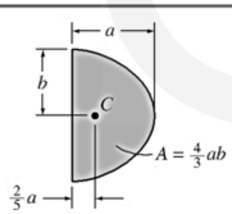

Exparabolic area



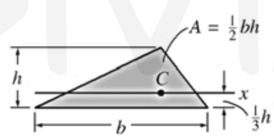

Rectangular area

$I_x = \frac{1}{12} bh^3$

$I_y = \frac{1}{12} hb^3$



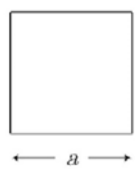

Parabolic area




Triangular area

$I_x = \frac{1}{36} bh^3$

### Square




Perimeter: $P = 4a$

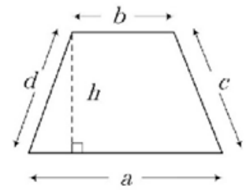
Area: $A = a^2$

### Rectangle


Perimeter: $P = 2(a+b)$

Area: $A = ab$

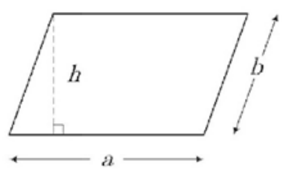
### Trapezoid




Perimeter: $P = a+b+c+d$

Area: $A = \left(\frac{a+b}{2}\right)h$

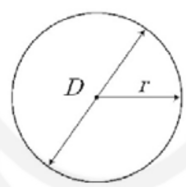
### Parallelogram




Perimeter: $P = 2(a+b)$

Area: $A = ah$

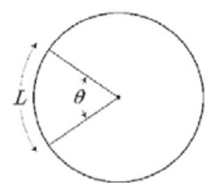
### Circle




Perimeter: $P = 2\pi r = \pi D$

Area: $A = \pi r^2 = \frac{\pi D^2}{4}$

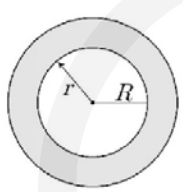
### Circular Sector




Arch Length: $L = \frac{\pi r \theta}{180^\circ}$

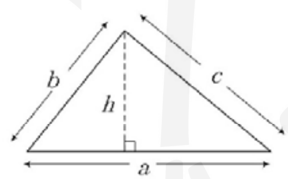
Sector Area: $A = \frac{\pi r^2 \theta}{360^\circ}$

### Circular Ring




Area: $A = \pi\left(R^2 - r^2\right)$

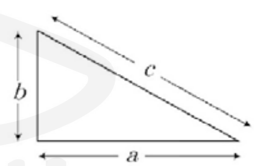
### Triangle




Perimeter: $P = a+b+c$

Area: $A = \frac{ah}{2}$

### Right Triangle



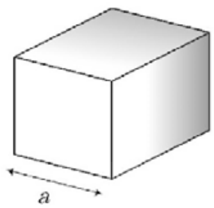

Perimeter: $P = a+b+c$

Area: $A = \frac{ah}{2}$

Pythagorean theorem:

$$c^2 = a^2 + b^2$$

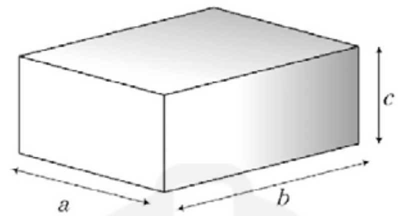
**Cube**




Surface Area: $A = 6a^2$

Volume: $V = a^3$

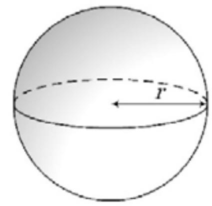
**Rectangular Solid**




Area: $A = 2(ab + ac + bc)$

Volume: $V = abc$

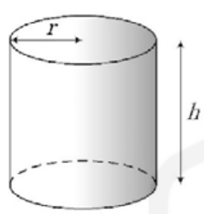
**Sphere**




Surface Area:
$A = 4\pi r^2$

Volume:
$V = \dfrac{4\pi r^3}{3}$

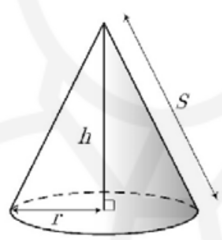
**Cylinder**




Surface Area:
$A = 2\pi r(r + h)$

Volume: $V = \pi r^2 h$

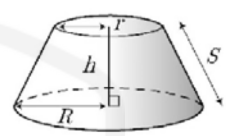
**Right Cone**




Surface Area:
$A = \pi r(r + S)$
$S = \sqrt{r^2 + h^2}$

Volume: $V = \dfrac{\pi r^2 h}{3}$

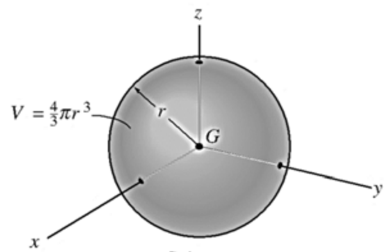
**Frustum of a Cone**




Area:
$A = \pi\left[Q(R - r) + (R^2 - r^2) + RS\right]$
$Q = \sqrt{r^2 + \left(\dfrac{Hr}{R - r}\right)^2}$
$S = \sqrt{(R - r)^2 + H^2}$

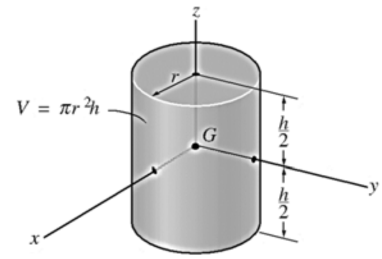
Volume: $V = \dfrac{\pi h}{3}\left(r^2 + rR + R\right)$

## 7.4 Center of Gravity and Mass Moment of Inertia of Homogenous Solids:



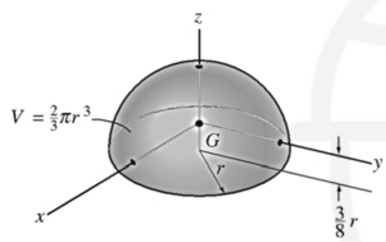

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$



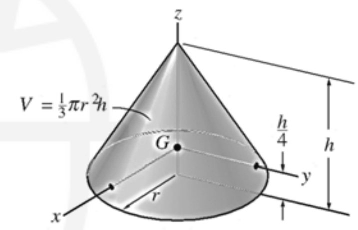

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$



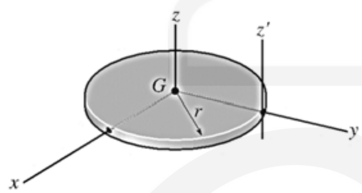

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



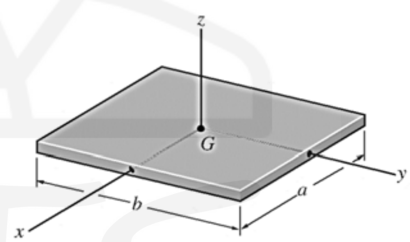

Cone

$I_{xx} = I_{yy} = \frac{3}{80}m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$



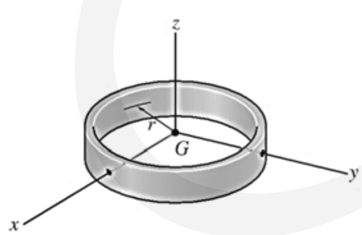

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2}mr^2$



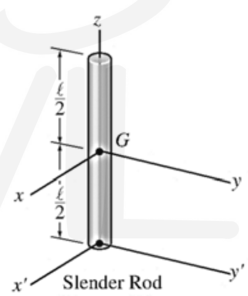

Thin plate

$I_{xx} = \frac{1}{12}mb^2 \quad I_{yy} = \frac{1}{12}ma^2 \quad I_{zz} = \frac{1}{12}m(a^2 + b^2)$




Thin ring

$I_{xx} = I_{yy} = \frac{1}{2}mr^2 \quad I_{zz} = mr^2$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12}m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3}m\ell^2 \quad I_{z'z'} = 0$

## 7.5 Fundamental Equations of Statics:

**Cartesian Vector**

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

*Magnitude*

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

*Directions*

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$
$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

**Dot Product**

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

**Cross Product**

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

**Cartesian Position Vector**

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

**Cartesian Force Vector**

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

**Moment of a Force**

$$M_o = Fd$$
$$\mathbf{M}_o = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

**Moment of a Force About a Specified Axis**

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

**Simplification of a Force and Couple System**

$$\mathbf{F}_R = \Sigma\mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

**Equilibrium**

*Particle*

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$

*Rigid Body-Two Dimensions*

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0$$

*Rigid Body-Three Dimensions*

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$
$$\Sigma M_{x'} = 0, \Sigma M_{y'} = 0, \Sigma M_{z'} = 0$$

**Friction**

*Static (maximum)* $F_s = \mu_s N$

*Kinetic* $F_k = \mu_k N$

**Center of Gravity**

*Particles or Discrete Parts*

$$\bar{r} = \frac{\Sigma\tilde{r}W}{\Sigma W}$$

*Body*

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

**Area and Mass Moments of Inertia**

$$I = \int r^2\,dA \qquad I = \int r^2\,dm$$

*Parallel-Axis Theorem*

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

*Radius of Gyration*

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

**Virtual Work**

$$\delta U = 0$$

## 7.6 SI Prefixes:

| *Multiple* | *Exponential Form* | *Prefix* | *SI Symbol* |
|---|---|---|---|
| 1 000 000 000 | $10^9$ | giga | G |
| 1 000 000 | $10^6$ | mega | M |
| 1 000 | $10^3$ | kilo | k |
| *Submultiple* | | | |
| 0.001 | $10^{-3}$ | milli | m |
| 0.000 001 | $10^{-6}$ | micro | $\mu$ |
| 0.000 000 001 | $10^{-9}$ | nano | n |

## 7.7 Conversion Factors (FPS) to (SI)

| *Quantity* | *Unit of Measurement (FPS)* | *Equals* | *Unit of Measurement (SI)* |
|---|---|---|---|
| Force | lb | | 4.448 N |
| Mass | slug | | 14.59 kg |
| Length | ft | | 0.3048 m |

## 7.8 Conversion Factors (FPS):

1 ft = 12 in. (inches)
1 mi. (mile) = 5280 ft
1 kip (kilopound) = 1000 lb
1 ton = 2000 lb